OSCAR II VISIBILITY ALGORITHM
Technical overview for OAA/JICPAR Working Party

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Introduction:

The proposed algorithm is based on a model of the visual behaviour of a driver (observer or O) passing a poster panel, and incorporates an account of the visual input received by O arising from the panel, and uses visibility parameters for O drawn from the experiment conducted for OAA at Birkbeck College in March 1995. The panel is assumed to be seen in daylight without artificial illumination. A brief technical description of the algorithm, for possible implementation as a computer program, is included as Annex 1.

An algorithm for specifying panel visibility was developed on the understanding of what the database of poster site information currently includes, and what other sources will provide in the near to immediate future. Accordingly certain site "parameters" are expected to be supplied in order to compute panel visibility. Four parameters are required - vehicle speed; distance of panel when first capable of being seen by driver; distance of leading edge of panel from the centre of the left-hand lane of the road; the size of panel (i.e., 6 sheet, 48 sheet, etc.). In an extension of the model, an additional parameter, the angle of deflection of the panel from a head-on position, is also required. The model assumes that the "minimum visibility distance[1]" (MVD) constitutes the *maximum* distance from the panel for computing contributions to visibility "hit rates". That is, hits can only begin to be accumulated when the panel is viewed at a distance less than its MVD.

Two versions of the model are presented, however, a number of refinements (see *Caveats*) may be needed before a final version is recommended. The versions outlined below vary in relation to how the panel geometry enters the visibility calculations. In one case what is used as the size parameter in the estimation process is the diameter of a disc equal in area to the panel. In the other the size parameter is the width of the panel. There are arguments for both. What perhaps ought to be decisive is which version the more successfully converges on the data from the experiment - these data are, after all, the foundation of the model, and the best estimate to hand of what drivers do when confronted by a roadside poster panel.

Caveats

Aside from this choice, there are some outstanding issues to be settled. For instance, it needs to be decided how far the model can be adjusted to accommodate any difference between viewing with or without illumination, and any contribution due to "clutter". Essentially clutter refers to irrelevant visual information, and is assumed to affect the visibility of a target. Ideally it needs to be measured independently. Any index of clutter should clearly reflect the presence of salient objects, surfaces, illumination sources, and so forth. At present no measure of clutter based on visual analysis is available. In practice it is assumed that clutter will be found to vary according to the type of site, as classified in the existing OSCAR database - shopping, arterial and residential. These are assumed to be ordered according to the amount of competing irrelevant visual information. The choice of

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[1] The terminology in force at the time of writing this report was confused. The idea of a "maximum visibility distance" is intuitively more understandable and this has become the accepted conceptual label.

site classification as an indicator of clutter was a common-sense decision, but it could be readily checked by a simple subjective rating study. For the moment, it is of particular interest that the hit rates observed in the experiment decreased as clutter increased, as would be expected. There are other aspects that need further analysis, consideration and/or experimentation; it will be remembered that one major issue sidelined for the present is the question of an account of pedestrian visibility. For practical purposes, however, a sensible end-point has been reached with the detailed specification of an applicable model. .

As agreed by the Steering Committee, the contribution of the poster content is not taken into account. What is given by the model is a "representative" position, defined by how the Os in the experiment looked at the panels that appeared in the photographs used. To the extent that creative content varied, so the behaviour that was elicited may well have varied, and the relationships that were estimated in the experiment (e.g., as to how visibility varied with eccentricity) will be averages over the set of posters used. A successful poster will attract more fixations on a panel than the model predicts.

Computation of a visibility "netting-down" factor

The computation of visibility proceeds over a number of steps, summarized in Annex 1. Step 1 entails the specification of the input parameters mentioned above. Steps 2 and 3 will be described in detail, so that the structure of the model can be understood. However, a simplified version of Step 3 will be explained first. Step 4 refers to some variations and refinements of the model that need to be evaluated before possible implementation. Step 5 is a generalization of the model to incorporate the contribution of panel deflection.

Preliminaries

The probability model used in Step 3 requires the cumulation of what may be thought of as momentary or elementary contributions to the overall hit rate, arising from the target object seen by O as if in a finely graded series of snapshots during the approach to the panel. It is obvious that the image of the object at the O's eye (if steadily fixating a point ahead) will increase in size, and it will move out to the edge of the eye, as the distance between O and panel decreases. Plainly the hit rate is likely to depend on size (and eccentricity), and so this information will need to be incorporated. Indeed the trajectory of the image across a stationary retina is dealt with later. For the moment, let us consider that the image of the panel does not change, and that the O is equally likely to fixate the panel at any moment during his/her passage past the panel. Suppose that the instantaneous probability of fixating the panel for an instant of duration τ is p. This might be measured, for example, by the proportion of a large population of individuals who fixate the panel in the course of an exposure duration τ. Clearly the size of τ will matter, and again this is returned to later.

Consider the entire series of momentary opportunities to see the panel as O approaches and passes it. What is the probability of seeing (i.e., fixating) the panel, aggregated over the total viewing interval? In the course of the first moment, the probability of a hit is p, and the probability of a miss, which for convenience we call q is therefore equal to 1-p. The probability of having made a hit before the second moment begins is therefore p.

The probability that a hit is made in the second moment is pq (the instantaneous probability of fixating the panel x the probability of not having fixated it before the start of the second moment). Hence the probability of a hit so far (before the start of the third moment) is p + pq. The probability of a miss so far is 1 - p - pq or q^2 . Similarly the probability that a hit is made during the third moment is pq^2, and the probability of a miss before the start of the fourth moment is q^3. By similar reasoning, the cumulative probability of a hit by the jth moment is $1 - q^j$ or $1- (1 - p)^j$. This simple model is the basis of the more general account given in Step 3. It was found empirically (by trying different values of what counts as a "moment"), that changing the time interval (for instance, from 0.1 to 0.3 second), made barely any difference to the probability calculations, certainly at the level of precision of the measures made in the experiment or otherwise available.

What is now needed as preparation for Step 3 is an account of the image trajectory (a geometrical problem) and a link between the O's viewing behaviour and some relevant properties of the image. The second of these requirements is met by the experimental data from the visibility experiment.

The visibility algorithm

Step 1. This will entail the input of the site parameters required by the visibility algorithm: vehicle speed (V mph); distance from panel when first potentially visible (D); distance of leading edge of panel from the centre of the driver's lane (L); size of panel (S). The last of these is converted to a convenient form in the process of computation. The value of D is the unobstructed distance to the panel. Separate contributions may be accumulated for multiple lane and two-way flow conditions, given that the panel is still potentially visible.

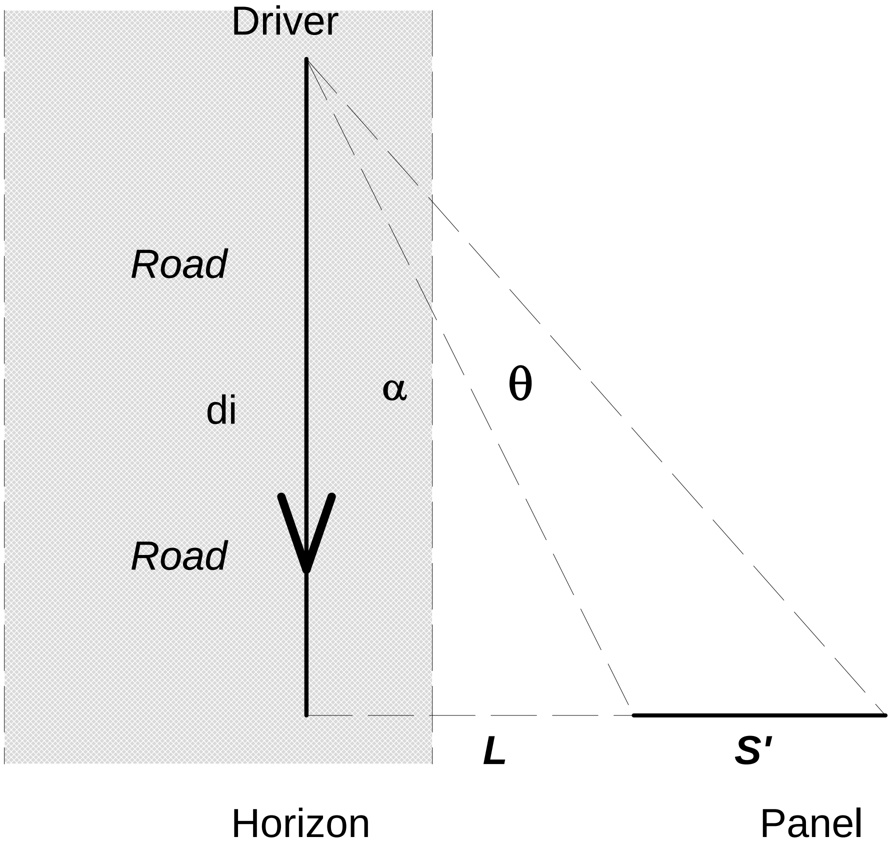
Step 2. (i) The viewing interval (T) can be obtained from the vehicle speed (V) and distance from the panel (D) when first capable of being seen (subject to the MVD criterion for the panel). This interval is divided into suitable units of duration τ seconds (the unit in the actual computations is one-tenth of a second). The distance d travelled in a duration of τ is obtained.

The trajectory and probability calculations are done for the series of times $t_i = T, T-\tau, T-2\tau, T-3\tau$, etc., until the panel is passed (at t = 0).

The distances from the panel d_i corresponding to these times are calculated: D, D-d, D-2d, D-3d, etc., until the panel is passed (at distance = zero).

(ii) From a simple geometrical representation, the angles θ_i subtended by the panel are calculated using $\theta_i = \arctan (L + S') /d_i - \arctan L /d_i$ (equation 1). The values required may be seen in Figure 1.

Figure 1: Basic geometry of panel site



The angles θ and α vary as the distance from the point at which the vehicle passes the panel decreases. It will be obvious from Figure 1 that α decreases, but the behaviour of θ is not so obvious. In fact in many real circumstances θ will increase for the most part of the approach and will finally and rapidly decrease as the panel is passed; in others it will decline gradually over the entire interval. Figure 2 illustrates the effects for a representative set of panel sizes/positions. These cover the range used in the visibility experiment; it should be remembered that all photographs for the experiment were taken at 30 metres (the MVD for 6 sheets). It should also be noted that as the driver travels towards the panel there is also a visual shape change, which has not so far been included in the modelling.

Note that the requirement for a visibility netting down factor has been generalized so that it will accommodate all distances for panels at which they are potentially visible (i.e., less than their MVD[2]). Accordingly for 48 and 96 sheet panels, the model has to apply for distances from 100 metres and below, and for 6 sheets from 30 metres and below. It will be appreciated that the geometrical space of alternatives therefore extends 70 metres back from the 30 metre point that is the origin of the axis in Figure 2, and that the only points sampled in the corresponding behavioural space (i.e., in the visibility experiment) are those at the 30 metre point.

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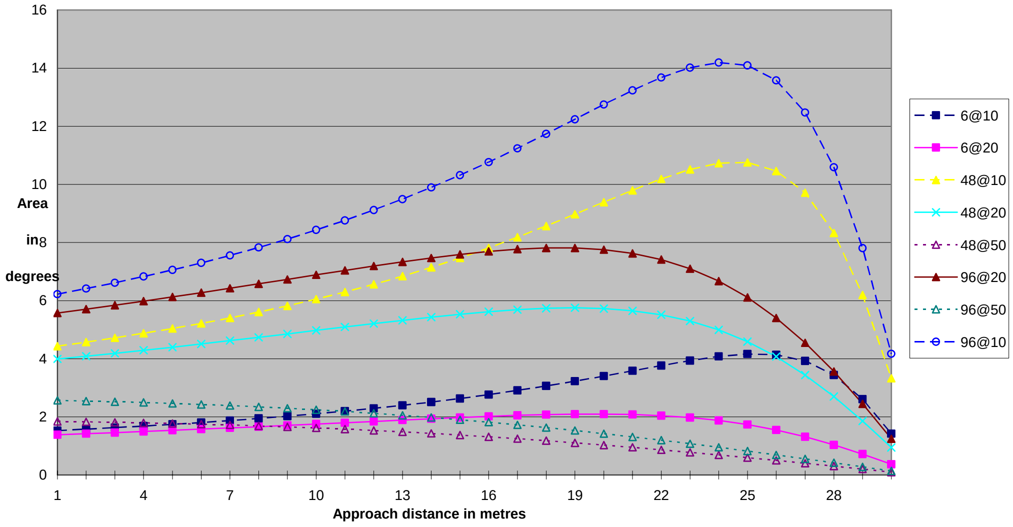
[2] The MVD values specified in this paper were those currently used in the original OSCAR system. An empirically based revision of MVDs was undertaken in a later paper in this series (Barber and Dickenson, 1997).

Figure 2: Subtended area as a function of approach distance for a sample of panel formats



(iii) The link between the geometry and behaviour - and hence the visibility experiment - becomes crucial at this point. In the experiment the "drivers" were given a 6 second view of each scene, and "percentage hit rates" for each image condition (a combination of size, eccentricity and clutter) were calculated by aggregating across all subjects in the experiment, and across all relevant images. The detailed evidence from the experiment will be reported elsewhere. For present purposes interest lies in any relation between hit rate and the geometrical properties of the panel images as presented to the driver-subjects. It is planned in due course to devise a comprehensive visibility model (a response surface reflecting significant contributions of all the factors in the experiment), but the analysis for this is not yet complete. In any event a relatively straightforward solution to the problem of the geometry-behaviour link was discovered since a strong positive correlation (r = 0.93) between observed aggregate hit rates and panel areas as specified by equation 1 above was found. On the face of it, a linear regression could be used to estimate hit rate given panel area. However, for areas above a certain size, this will yield hit rates in excess of 100% (possibly an advertiser's dream, but an impossible one) and so an alternative solution was sought. Many were considered (including that of Cole and Hughes, 1986); the eventual solution involved nonlinear regression and an exponential function $p_i = 100\ (1 - e^{k\theta i})$ .... (equation 2). This has the benefit of being bounded by 0 and 100%, and when optimized by a least squares regression solution, there is a strong positive correlation between hit rates and values given by the function. The value of k is estimated from the observed hit rate data.

This function is incorporated into the algorithm. Two versions are presented, one assuming that size may be represented by the visual angle subtended by a disc equal in area to the panel. This has the merit of using area information. The second is to represent size by the width of the panel. There are arguments in favour of both versions. Although there may well be overwhelming pragmatic considerations, it is arguable that what should be decisive technically is which of the two gives the better account of the data from the visibility experiment.

Step 3. A generalization of the probability model outlined above is applied to the areas produced by equation 2. The important difference between the simple model and the revised version is that p varies with area, as the panel is approached, even though the basic step-by-step mechanics of the model are the same. The cumulative hit rate over the approach distance is given by $\Sigma p_N = \Sigma p_{N-1} + q_{N-1}\ p_N$. This entails an iterative calculation, in practice this is for each of the N one-tenth second time intervals occupied in the approach; it is tantamount to the integration of a probability density function that varies with object area, over the trajectory of areas traversed by the object on a notional driver's eye (looking down the road ahead) as the driver approaches and finally passes the object.

Figure 3 shows how hit rate accumulates over the approach interval for some typical panel configurations, assuming a vehicle travelling at a constant 12 mph (note that it covers a 30 metre approach distance in less than 6 seconds). The model will report the final value in each case.

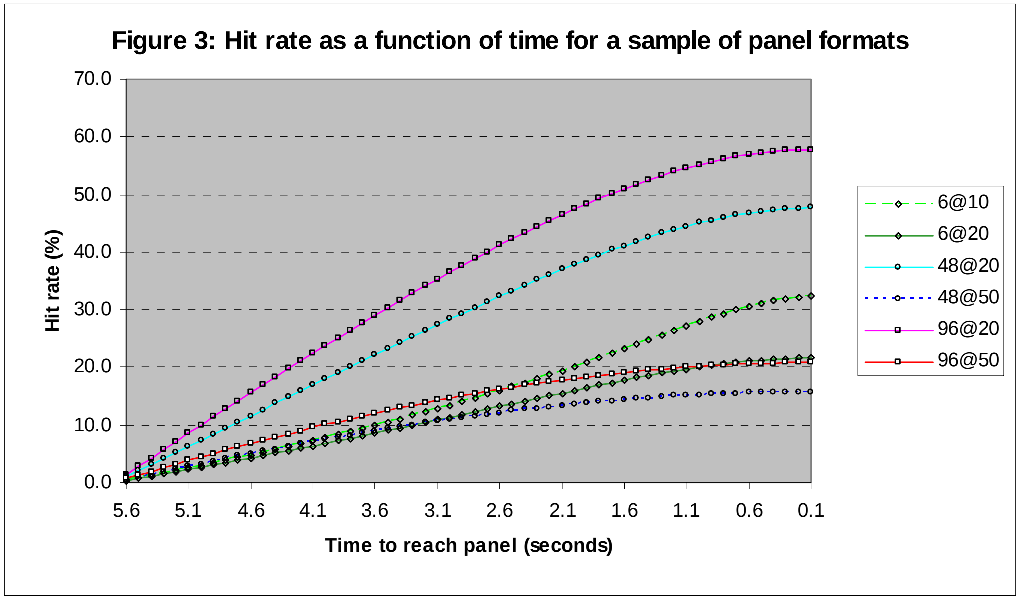


Table 1 provides examples of theoretical hit rates for the MVDs for 6, 48 and 96 sheets, for a variety of vehicle speeds, and for two versions of the model in which panel size is accounted for by different geometrical measures (the "width" and the "disc" versions). The eccentricities of 10, 20 and 50 degrees correspond to panels set back from the middle of the nearside lane of 5.3, 10.9 and 35.8 metres respectively, the angles being used for reference purposes relative to the visibility experiment which entailed posters set back by these distances and photographed at 30 metres. A visibility model would not be acceptable if hit rates did not decline with eccentricity, increase with panel size, and decrease as vehicle speed increases; both versions of the current model pass this basic test. Of more interest in practice will be the form of these relationships. It will be noted, for instance, that the hit rate "cost" of increased vehicle speed is less in relative terms for a less eccentric panel (e.g., the hit rate for a 96 sheet at 20° falls by 20% as speed increases from 8 to 20 mph, whereas for the same panel at 50° the fall is 32%). These kinds of comparisons may be of interest and can be easily obtained from the model.

Table 1: Theoretical percent hit rates for different combinations of panel size, eccentricity and vehicle speed

WIDTH VERSION*

6 sheet

		Eccentricity in degrees		
	Speed (mph)	10	20	50
Potential	8	25.6	16.8	n/a
Sighting	12	17.9	11.6	n/a
At	16	13.8	8.8	n/a
30m	20	11.2	7.2	n/a

48 sheet

		Eccentricity in degrees		
	Speed (mph)	10	20	50
Potential	8	90.0	83.7	60.4
Sighting	12	78.4	70.1	46.1
At	16	68.3	59.6	37.1
100m	20	60.2	51.6	31.0

96 sheet

		Eccentricity in degrees		
	Speed (mph)	10	20	50
Potential	8	98.4	96.4	82.5
Sighting	12	93.6	89.1	68.7
At	16	87.5	81.0	58.2
100m	20	80.8	73.5	50.3

DISC VERSION*

6 sheet

		Eccentricity in degrees		
	Speed (mph)	10	20	50
Potential	8	44.3	30.6	n/a
Sighting	12	32.3	21.7	n/a
At	16	25.4	16.8	n/a
30m	20	20.9	13.7	n/a

48 sheet

		Eccentricity in degrees		
	Speed (mph)	10	20	50
Potential	8	93.6	88.4	66.4
Sighting	12	84.0	76.2	51.7
At	16	74.6	65.9	42.1
100m	20	66.8	57.8	35.4

96 sheet

		Eccentricity in degrees		
	Speed (mph)	10	20	50
Potential	8	97.6	94.8	77.9
Sighting	12	91.7	86.0	63.5
At	16	84.6	77.2	53.1
100m	20	77.6	69.3	45.4

* The two versions of the model differ in how they treat panel size. In the Width model, the horizontal angle subtended by the panel is used, whereas in the Disc model, it is the angle subtended by the diameter of a disc of equal area to the panel.

Table 2 also gives hit rates for approach distances equal to half the MVD in each case; this can be used to illustrate the relative gain from having an obstructed view of the panel for the first half of the approach.

Table 2(a): Theoretical hit rates (disc version) as a function of panel size, eccentricity and distance (no eccentricity cut-off applied; vehicle speed 12 mph)

PANEL TYPE	APPROACH DISTANCE	ECCENTRICITY* (degrees) 10	20	30	50
6 SHEET	15 m	21.2	11.2	n/c	n/c
48 SHEET	50 m	n/c	62	50.9	35.2
96 SHEET	50 m	n/c	73	62.1	48.2
6 SHEET	30 m	32.3	21.7	n/c	n/c
48 SHEET	100 m	n/c	76.2	68.7	51.7
96 SHEET	100 m	n/c	86	79.8	63.5

Table 2(b): Theoretical hit rates (disc version) as a function of panel size, eccentricity and distance (65 degree cut-off applied; vehicle speed 12 mph)

PANEL TYPE	APPROACH DISTANCE	ECCENTRICITY* (degrees) 10	20	30	50
6 SHEET	15 m	19.3	9.1	n/c	n/c
48 SHEET	50 m	n/c	58.7	46.8	27.7
96 SHEET	50 m	n/c	69	57.3	37.5
6 SHEET	30 m	30.6	19.7	n/c	n/c
48 SHEET	100 m	n/c	74	66.2	48.1
96 SHEET	100 m	n/c	84.4	77.7	59.3

* These eccentricities refer to the angle subtended by the leading edge of the panel viewed from 30 metres
n/c = not calculated

Step 4. No restriction has so far been imposed on the eccentricity of the panel, nor on the size of the areas that may contribute to hit rates. This should be easy to include in the algorithm. It may be best to trial the model empirically with various restrictions (e.g., limiting the eccentricity of an object to 65° to either side, which is roughly about the angle at which head movements are essential for an object to be seen). Clutter may also be easily incorporated in a final version, but illumination may require something more radical.

Some examples of the effect of restricting the eccentricity to 65° are given in Table 2. Values in Table 2(b) were returned by the model with this restriction in place. While all hit rates are automatically reduced, those for large eccentricities are more affected than those for small eccentricities. This extra netting down of hit rates has the advantage of producing an eccentricity gradient that is closer to the data from the experiment. For a 48 sheet at 50 metres approached at 12 mph, the hit rate difference for panels at 20° and 50° is 27% without and 31% with the 65° cut-off. Similarly the difference for two 96 sheets at the same eccentricities and under the same conditions is 25% and 32% respectively. This would seem to be an effective and justifiable way of increasing the weight of eccentricity. The Table shows that the differences in these two cases are almost eradicated if the approach distance is 100 metres, as the interval (i.e., the last few metres of approach) when the panel becomes very eccentrically placed is proportionately much smaller relative to the 50 metres case.

Step 5 has been added to allow for *deflected panels*. The reasoning is that the model deals with visual areas, and deflection affects visual area, so the geometry should accommodate this aspect of site geometry too. The model appears to generate sensible data. In this report "deflection" refers to the angle through which a panel is turned in the direction of the oncoming traffic. Thus the 0° and 90° deflections are what are normally referred to as head-on and parallel panels respectively. All previous Tables therefore are for the 0° case. Table 3 samples the intervening values, assumes a vehicle speed of 12 mph, and a range of other representative conditions. A revised version of the geometry for the simple panel site is included as Figure 4.

TABLE 3a: Theoretical hit rates as a function of panel deflection and other factors - no eccentricity cut-off applied (vehicle speed 12 mph)

PANEL TYPE	**DEFLECTION (degrees)**					**DEFLECTION (degrees)**				
6 SHEETS	**0**	**22.5**	**45**	**67.5**	**90**	**0**	**22.5**	**45**	**67.5**	**90**
ECCENTRICITY	APPROACH DISTANCE - 15 metres					APPROACH DISTANCE - 30 metres				
10 deg	21	28	30	29	22	32	38	39	36	25
20 deg	11	17	20	21	18	22	28	31	30	24
48 SHEETS	APPROACH DISTANCE - 50 metres					APPROACH DISTANCE - 100 metres				
20 deg	62	71	72	68	56	76	82	82	76	59
30 deg	51	62	65	63	54	69	76	77	73	59
50 deg	30	43	49	50	46	52	63	67	65	56
96 SHEETS	APPROACH DISTANCE - 50 metres					APPROACH DISTANCE - 100 metres				
20 deg	73	81	82	78	66	86	90	90	85	69
30 deg	62	73	76	74	64	80	86	87	83	70
50 deg	39	54	60	61	56	64	75	78	76	67

TABLE 3b: Theoretical hit rates as a function of panel deflection and other factors - 65 degree eccentricity cut-off applied (vehicle speed 12 mph)

PANEL TYPE	**DEFLECTION (degrees)**					**DEFLECTION (degrees)**				
6 SHEETS	**0**	**22.5**	**45**	**67.5**	**90**	**0**	**22.5**	**45**	**67.5**	**90**
ECCENTRICITY	APPROACH DISTANCE - 15 metres					APPROACH DISTANCE - 30 metres				
10 deg	19	23	24	21	16	31	34	34	28	19
20 deg	9	12	13	13	10	20	24	25	22	17
48 SHEETS	APPROACH DISTANCE - 50 metres					APPROACH DISTANCE - 100 metres				
20 deg	59	64	63	56	42	74	77	75	66	47
30 deg	47	53	54	50	40	66	71	69	63	46
50 deg	24	31	33	32	27	48	55	56	52	41
96 SHEETS	APPROACH DISTANCE - 50 metres					APPROACH DISTANCE - 100 metres				
20 deg	69	74	72	66	51	84	86	85	77	57
30 deg	57	63	63	60	48	77	81	80	74	57
50 deg	31	38	40	39	34	59	66	67	63	51

It is obvious that for any specific approach distance there is an angle through which a head-on panel by the roadside would need to be turned in order to maximize the area it presents to an observer at that distance. Hence for panels set back from the road, there will always be a better position than head-on (0°). The model agrees with this, as can be seen in Table 3. Hit rate depends on set back from the road (eccentricity in current parlance). The Table thus indicates the relative advantage of different degrees of deflection. The Table is also useful for providing some evidence on the controversial matter of parallel panels at the roadside and viewed from afar. The logic is as follows. Consider the contrast between the 50 m and 100 m data for the 48 sheet panel at 20° (the least eccentric condition run for this table). The panel will be at its "thinnest" when encountered at 100 m, so the difference between the 50 m and 100 m hit rates is attributable to the contribution to hit rate from the panel at its thinnest. From Table 3(b), the first 50 m of a 100 m approach will contribute only 5% of the total hit rate of 47%. A similar comparison holds for 96 sheets positioned at 20°. Arguably the model treats the contribution of distantly viewed parallel panels with due weight. It does so by weighting contributions by visual area.

Figure 4: Geometry of panel site with deflected panel

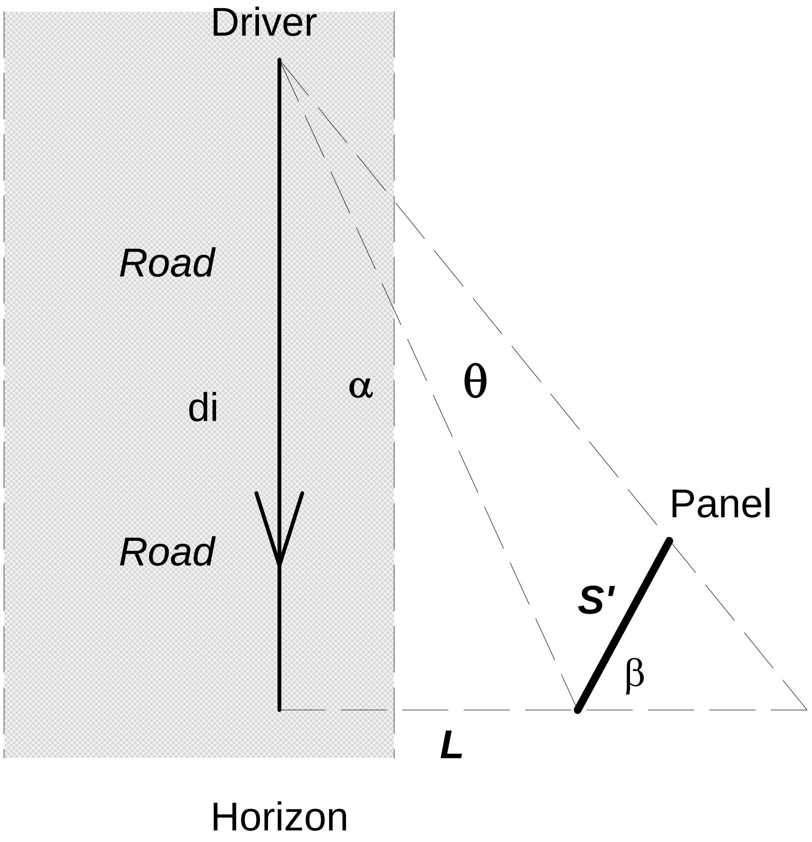
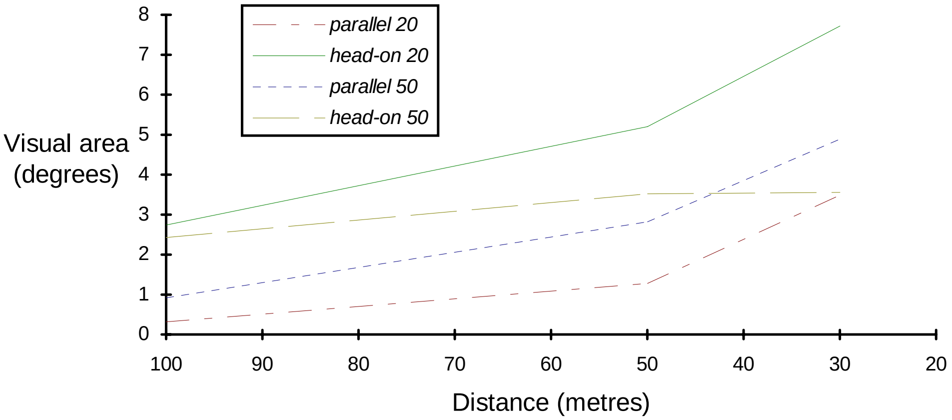


Figure 5 may be instructive in this connection; it shows the angles subtended by head-on and parallel 48 sheets for three distances and two eccentricities. At 100 m the less eccentric (20°) parallel panel offers much less than the comparable head-on panel for the observer to spot. At 30 m the parallel panel is still at a disadvantage but they are considerably less unequal. The balance actually reverses in the case of the more eccentric pair of panels (50°) when viewed at 30 m, and the parallel panel then offers a bigger target for the observer.

Between 100 and 50 m the rank ordering is head-on 20 > head-on 50 > parallel 50 > parallel 20, but as the panel is approached this ordering changes, as the parallel aspect begins to come into its own. In any event, the model appears to convey the hit rate effects and to accommodate the various contingencies of concern rather successfully.

Figure 5:
Visual area of head-on and parallel 48 sheet panels



As to *clutter*, the best evidence to hand is from the visibility experiment, which supplied hit rates for the three types of sites. The contribution of clutter was not great and could be accommodated by netting down from the model values, by indexing the panel site type to the experimental hit rates. Until a fuller analysis can be carried out (to check whether there are significant interactions involving clutter) it would be reasonable to assume a simple contribution from clutter. If factors of 1.00, 0.95 and 0.90 (for residential, arterial and shopping categories respectively) are applied the model's arithmetic should continue to make sense.

It remains to list some of the *unfinished business*. Many queries have been raised and most have been dealt with.

- *panel height* (i.e., the height of the lower edge above the horizontal) may best be accommodated for the present by adjusting the set back distance using Pythagoras Theorem! It is possible that the scanning patterns adopted by drivers may penalize elevated panels, and there may be some data from the visibility experiment to make a rough test of this. But there may be less clutter in the vicinity of such panels, so it is impossible at present to know one way or another. In any event the variation in height is rather small relative to that in horizontal displacement, so for the time being it would seem safest to assume there is no bias, and allow the model to account for the extra displacement to the side as proposed, using the ancient Greek authority.

- the *traffic light effect* has been identified as a problem. Panels may be sited at traffic lights or roundabouts where there is a long(er) opportunity to view. The model could easily accommodate this by a mixture of the usual dynamic model and a special case static model. The proportion of traffic affected would need to be specified, as well as the time and average distance from the panel when at a standstill.

- it should be remembered that the model is built on the data for first hits. Priority has been given to the matter of providing a visibility model, and not to a comprehensive analysis of the data. In fact there is an abundance of data yet to be analysed from the visibility experiment, including the repeat hit rate (i.e., how often observers take more then one look at the panel), and the amount of time spent viewing the panel. Further analysis would seem to be advisable to check the robustness of the model.

REFERENCES:

COLE, B.L., and HUGHES, P.K. (1984). A field trial of attention and search conspicuity. *Human Factors*, **26**, 299-313.

ANNEX 1:

OSCAR 2: VISIBILITY ALGORITHM

Assume all linear measurements are in metres, though speed may be expected to be supplied (by user) in mph. Angular measures are in degrees.

1. Input parameters:
- vehicle speed (V mph)
- distance from panel at first sight (D)
- distance of leading edge of panel from centre of left-hand lane (L)
- size of panel (S)
- deflection angle (β)

2. (i) Compute passage time (T) in seconds from V/D, and hence distance travelled in metres in 0.1 second (d).

   (ii) Compute d_i, series of distances corresponding to D, D-d, D-2d, ..., 0 for times t_i = T, T-0.1, T-0.2, ..., 0.

   (iii) Compute θ_i , series of panel "areas", i.e., angles subtended by panel at distances d_i using:

   $$\theta_i = \arctan (L + S') /d_i - \arctan L /d_i \qquad \text{........ Equation 1}$$

   where S' is the width in metres of the panel whose size is S (use a lookup table for conversion?), or S' is the diameter of the equivalent disc.

   (iv) Compute p_i, series of "momentary" p values (hit rates) for areas given by θ_i using:

   $$p_i = 100 (1 - e^{k\theta i}) \qquad \text{........ Equation 2}$$

   and $p_0 = 0$.

   If the width version of S' is used, then k = - 0.000926; but if the disc version of S' is used, then k = - 0.001348.

3. Find the cumulative hit rate Σp_N using

   $$\Sigma p_N = \Sigma p_{N-1} + q_{N-1} \; p_N \qquad \text{........ Equation 3}$$

   where $q_0 = 1$ and N = INT (10T)

4. Apply restrictions on the cumulation if arctan (L /d_i) exceeds (say) 65° , by setting $p_i = 0$ for this and all succeeding angles. A restriction may also be needed to deal with very small areas, but this may be adequately covered by the restriction on the leading edge angle.

5. Deflection from the head-on position requires that Equation 1 is replaced by the following:

$$\theta_i = \arctan (L + S' \cos \beta) /(d_i - S' \sin \beta) - \arctan L /d_i \qquad \text{........ Equation 4}$$

where β is the angle of deflection, and the eventual cumulation is for D, D-d, D-2d, ..., D-S' sin β. The eccentricity cut-off would also apply.

Paul Barber for OAA/August 1995

FOOTNOTES ADDED: FEBRUARY 2006